Bibliography

Alomar, B. O. (2007). Personal and family factors as predictors of pupils' mathematics achievement. *Psychological Reports, 101,* 259-269.

Anderson, S. E. (1990). Worldmath curriculum: Fighting Eurocentrism in mathematics. *Journal of Negro Education, 59,* 348-359.

Andrade, H., & Valtcheva, A. (2009). Promoting learning and achievement through self-assessment. *Theory Into Practice, 48,* 12-19.

Ashlock, R. (2009). *Error patterns in computation*. New Jersey: Prentice Hall.

Ball, D. L., & Hill, H. C. (2009). Measuring teacher quality in practice. In D. H. Gitomer (Ed.), *Measurement issues and assessment in teaching quality* (pp. 80-98). Thousand Oaks, CA: Sage.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59,* 389 – 407.

Barratta-Lorton, M. (1976). *Mathematics their way*. Menlo Park, CA: Addison Wesley Publishing Co.

Baroudi, Z. (2006). Easing students' transition to algebra. *Australian Mathematics Teacher, 62,* 28-33.

Battista, M. T. (2007). The development of geometric and spatial thinking. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 843-908). Reston, VA: National Council of Teachers of Mathematics.

Behr, M. (1980). How children view the equal sign. *Mathematics Teaching, 92,* 13-15.

Behr, M. J., Harel, G., Post, T. R., & Lesh, R. (1992). Rational number, ratio, and proportion. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 296-333). Reston, VA: National Council of Teachers of Mathematics.

Brownell, W. A. (1956). Meaning and skill: Maintaining the balance. *Arithmetic Teacher, 34,* 18-25.

Caldwell, J., Karp, K., & Bay-Williams, J. M., (2011). *Developing essential understanding of addition and subtraction for teaching mathematics in prekindergarten-grade 2*. Reston, VA: NCTM.

Carpenter, T., Fennema, E., Franke, M., Levi, L., & Emperson, S. (1999). *Children's mathematics: Cognitively guided instruction.* Portsmouth, NH: Heinemann.

Carraher, D. W., Martinez, M. V., & Schliemann, A. D. (2008) Early algebra and mathematical generalization. *ZDM Mathematics Education, 40,* 3-22.

Carraher, D. W., & Schliemann, A. D. (2007). Early algebra. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 669-706). Reston, VA: National Council of Teachers of Mathematics.

Carter, S. (2008). Disequilibrium and questioning in the primary classroom: Establishing routines that help students learn. *Teaching Children Mathematics, 15,* 134-137.

Clements, D. H., & Battista, M. T. (1992). Geometry and spatial reasoning. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 420-464). Reston, VA: National Council of Teachers of Mathematics.

Coleman, J. S. (1967a). '*Equality of educational opportunity,' reconsidered.* Washington, DC: U.S. Department of Health, Eduction & Welfare Office of Education. (ERIC Document Reproduction Service No. ED015893)

Coleman, J. S. (1967b). *The concept of 'Equality of educational opportunity.'* Washington, DC: U.S. Department of Health, Eduction & Welfare Office of Education. (ERIC Document Reproduction Service No. ED015157)

Coleman, J. S. (1968). *The evaluation of 'Equality of educational opportunity.'* Washington, DC: U.S. Department of Health, Education, & Welfare Office of Education. (ERIC Document

Reproduction Service No. ED026721)
Coleman, J. S., Campbell, E. Q., Hobson, C. J., McPartland, J., Mood, A. M., Weinfeld, F. D., & York, R. L. (1966). *Equality of educational opportunity*. Washington DC: U.S. Department of Health, Eduction & Welfare Office of Education. (ERIC Document Reproduction Service No. ED012275)
Crowley, M. L. (1987). The Van Hiele model of the development of geometric thought. In M. M. Lindquist & A. P. Shulte (Eds.), *Learning and teaching geometry, K-12: 1987 yearbook* (pp. 1-16). Reston, VA: National Council of Teachers of Mathematics.
Daggett, W. R. & Kruse, B. (1999). *Taming the educational dinosaur*. Rexford, NY: Leadership Press.
Daro, P., Mosher, F., & Corcoran, T. (2011). *Learning trajectories in mathematics: A foundation for standards, curriculum assessment and instruction*. Philadelphia, PA: Consortium for Policy Research in Education.
Dermitzaki, I., Leondari, A., & Goudas, M. (2009). Relations between young students' strategic behaviours, domain-specific self-concept, and performance in a problem-solving situation. *Learning and Instruction, 19,* 144-157.
Dougherty, B., Flores, A., Louis, E., & Sophian, C. (2010). *Developing essential understanding of number and numeration for teaching mathematics in prekindergarten-grade 2.* Reston, VA: NCTM.
Evans, J., & Tsatsaroni, A. (2000). Mathematics and its publics: Texts, contexts and users. *Social Epistemology, 14,* 55-68.
Fennell, F. (Ed.) (2011). *Achieving fluency in special education and mathematics.* Reston, VA: NCTM.
Fennema, E., & Sherman, J. (1977). Sex-related differences in mathematics achievement, spatial visualization, and affective factors. *American Educational Research Journal, 14*, 51-71.
Fennema, E., & Sherman, J. (1978). Sex related differences in mathematics achievement and related factors: A further study. *Journal for Research in Mathematics Education, 9,* 189-203.
Finn, C. E. (2005, January 26). Grading school standards: A tale of two studies. *National Review Online, 57*. Retrieved on June 29, 2009, from http://www.nationalreview.com/comment/finn200501260740.asp
Fosnot, C., & Dolk, M. (2001a). *Young mathematicians at work, constructing number sense, addition and subtraction.* Portsmouth, NH: Heinemann.
Fosnot, C., & Dolk, M. (2001b). *Young mathematicians at work, constructing multiplication and division.* Portsmouth, NH: Heinemann.
Fosnot, C., & Dolk, M. (2002). *Young mathematicians at work, constructing fractions, decimals and percents*. Portsmouth, NH: Heinemann.
Franklin, C., Kader, G., Mewborn, D. Moreno, J., Peck, R. Perry, M. & Scheaffer, R. (2005). *Guidelines for assessment and instruction in statistics education (GAISE Report).* Alexandria, VA: American Statistical Association.
Fuson, K. C. (1998). Pedagogical, mathematical, and real-world conceptual-support nets: A model for building children's multidigit domain knowledge. *Mathematical Cognition, 4,* 147-186.
Fuson, K. C., Kalchman, M. S., & Bransford, J. D. (2005). Mathematical understanding: An introduction. In M. S. Donovan & J. D. Bransford (Eds.), *How students learn: History, mathematics, and science in the classroom* (pp. 217-256). Washington DC: National Academies Press.
Fuson, K. C., Wearne, D., Hiebert, J. C., Murray, H. G., Human, P. G., Olivier, A. I., Carpenter, T. P., & Fennema, E. (1997). Children's conceptual structures for multidigit numbers and methods of multidigit addition and subtraction. *Journal for Research in Mathematics Education, 28,* 130-162.

Fuys, D. J., & Liebov, A. K. (1997). Concept learning in geometry. *Teaching Children Mathematics, 3,* 248-251.

Gardner, H. (1987). The theory of multiple intelligences. *Annals of Dyslexia, 37,* 19-35.

Gardner, H. (1989). Beyond a modular view of mind. In W. Damon (Ed.), *Child development today and tomorrow* (pp. 222-239). San Francisco, CA US: Jossey-Bass.

Gardner, H. (2009). The five minds for the future. *School Administrator, 66,* 16-21.

Gardner, H., & Hatch, T. (1989). Multiple intelligences go to school: Educational implications of the theory of multiple intelligences. *Educational Researcher, 18,* 4-9.

Gresham, G., Sloan, T., & Vinson, B. (1997). *Reducing mathematics anxiety in fourth grade "at-risk" students*. Orlando, FL: University of Central Florida. (ERIC Document Reproduction Service No. ED417931). Retrieved from http://www.eric.ed.gov/ERICWebPortal/search/detailmini.jsp?_nfpb=true&_&ERICExtSearch_SearchValue_0=ED417931&ERICExtSearch_SearchType_0=no&accno=ED417931

Griffin, S. (2005). Fostering the development of whole-number sense: Teaching mathematics in the primary grades. In M. S. Donovan & J. D. Bransford (Eds.), *How students learn: History, mathematics, and science in the classroom* (pp. 257-308). Washington DC: National Academies Press.

Hatch, T. C., & Gardner, H. (1986). From testing intelligence to assessing competencies: A pluralistic view of intellect. *Roeper Review, 8,* 147-150.

Hegedus, S. J., & Kaput, J. J. (2004, July). *An introduction to the profound potential of connected algebra activities: Issues of representation, engagement and pedagogy.* Paper presented at the annual meeting of the International Group for the Psychology of Mathematics Education, Bergin, Norway. (ERIC Document Reproduction Service Number ED489551). Retrieved from http://www.eric.ed.gov/ERICWebPortal/search/detailmini.jsp?_nfpb=true&_&ERICExtSearch_SearchValue_0=ED489551&ERICExtSearch_SearchType_0=no&accno=ED489551

Hiebert, J. (2003). What research says about the NCTM standards. In J.Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to "Principles and standards for school mathematics"* (pp. 5-23). Reston, VA: National Council of Teachers of Mathematics.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65-100). Reston, VA: National Council of Teachers of Mathematics.

Hiebert, J., Carpenter, T., Fennema, E., Fuson, K., Wearne, D., Murray, H., Oliver, A., & Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding*. Portsmouth, NH: Heinemann.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371-404). Reston, VA: National Council of Teachers of Mathematics.

Hiebert, J., Stigler, J. W., Jacobs, J. K., Givvin, K. B., Garnier, H., Smith, M. et al. (2005). Mathematics teaching in the United States today (and tomorrow): Results from the TIMSS 1999 video study. *Educational Evaluation and Policy Analysis, 27,* 111-132.

Hill, H. C. & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's mathematics professional development institutes. *Journal for Research in Mathematics Education, 35*.

Hill, H. C., & Ball, D. L. (2009). The curious—and crucial—case of mathematical knowledge for teaching. *Phi Delta Kappan, 91,* 68-71.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking Pedagogical Content Knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 29,* 372-400.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42,* 371-406.

Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *Elementary School Journal, 105,* 11-34.

Izsak, A. (2004). Students' coordination of knowledge when learning to model physical situations. *Cognition and Instruction, 22,* 81-128.

Jones, F. H. (1987). *Positive classroom discipline*. Columbus, OH: McGraw-Hill.

Kieran, C. (1980, August). *The interpretation of the equals sign symbol for an equivalent relation vs. an operator symbol.* Paper presented at the Fourth International Conference for the Psychology of Mathematics Education, Berkely, CA.

Kieran, C. (1989). The early learning of algebra: A structural perspective. In S. Wagner & C. Kieran (Eds.), *Research issues in the learning and teaching of algebra* (pp. 33-56). Hillsdale, NJ: Lawrence Erlbaum Associates.

Kieran, C. (1992). The learning and teaching of school algebra. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 390-419). Reston, VA: National Council of Teachers of Mathematics.

Kieran, C. (2007). Learning and teaching of algebra at the middle school through college levels: Building meaning for symbols and their manipulation. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 707-762). Reston, VA: National Council of Teachers of Mathematics.

Kieran, C. (2008). *What do students struggle with when first introduced to algebra symbols?* Reston, VA: National Council of Teachers of Mathematics. Retrieved on June 22, 2009, from http://www.nctm.org/uploadedFiles/Research_News_and_Advocacy/Research/Clips_and_Briefs/Brief%20-%20What%20Can%20We%20Learn.pdf

Kilpatrick, J., Swafford, J., & Findell, B. (2001). Adding it up: Helping children learn mathematics. Washington, DC: National Academy Press. Retrieved from http://www.nap.edu/catalog.php?record_id=9822#toc

Küchemann, D. (1978). Children's understanding of numerical variables. *Mathematics in School, 9,* 23-26.

Lamon, S. J. (1999). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers.* Philadelphia: Lawrence Erlbaum Associates.

Lamon, S. J. (2007). Rational numbers and proportional reasoning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 629-667). Reston, VA: National Council of Teachers of Mathematics.

Lehrer, R., Strom, D., & Confrey, J. (2002). Grounding metaphors and inscriptional resonance: Children's emerging understanding of mathematical similarity. *Cognition & Instruction, 20,* 359-398.

Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research, 60,* 1-64.

Ma, L. (1999). *Knowing and teaching elementary mathematics*. Hillsdale, NJ: Lawrence Erlbaum.

MacGregor, M., & Stacey, K. (1993). Cognitive models underlying students' formation of simple linear equations. *Journal for Research in Mathematics Education, 24,* 217-232.

MacGregor, M., & Stacey, K. (1997). Students' understanding of algebraic notation: 11-15. *Educational Studies in Mathematics, 33,* 1-19.

Maher, F. A., & Tetrault, M. K. T. (2001). *The feminist classroom* (2nd ed.). Oxford: Rowman & Littlefield.

Manouchehri, A., & Goodman, T. (1998). Mathematics curriculum reform and teachers: Understanding the connections. *Journal of Educational Research, 92,* 27-41.

Mirra, A. (2009). *Focus in PreK – 2: Teaching with curriculum focal points*. Reston, VA: NCTM.

Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for integrating technology in teacher knowledge. *Teachers College Record, 108*, 1017-1054.

Monaghan, F. (2000). What difference does it make? Children's views of the differences between some quadrilaterals. *Educational Studies in Mathematics, 42*, 179-196.

Moomaw, S., Carr, V., Boat, M., & Barnett, D. (2010). Preschoolers' number sense. *Teaching Children Mathematics*, 16 (6), 333-340.

Moss, J. (2005). Pipes, tubes, and beakers: New approaches to teaching the rational-number system. In M. S. Donovan & J. D. Bransford (Eds.), *How students learn: History, mathematics, and science in the classroom* (pp. 309-350). Washington D.C.: National Academies Press.

Moss, J., Beatty, R., Barkin, S., & Shillolo, G. (2008). "What is your theory? What is your rule?" Fourth graders build an understandingof functions through patterns and generalizing problems. In C. E. Greenes & R. Rubenstein (Eds.), *Algebra and algebraic thinking in school mathematics: Seventieth yearbook.* (pp. 155-168). Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2006). *Curriculum focal points: A quest for coherence*. Reston, VA: NCTM

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. Washington, D.C.: National Academy Press.

Niess, M. L. (2005). Preparing teachers to teach science and mathematics with technology: Developing a technology pedagogical content knowledge. *Teaching and Teacher Education: An International Journal of Research and Studies, 21*, 509-523.

Niess, M. L. (2011). Teacher knowledge for teaching with technology: A TPACK lens. In R. N. Ronau, C. R. Rakes, & M. L. Niess (Eds.), *Educational technology, teacher knowledge, and classroom impact: A research handbook on frameworks and approaches* (pp. 1-15). Hershey, PA: IGI Global. DOI: 10.4018/978-1-60960-750-0.ch001

Niess, M. L., Ronau, R. N., Shafer, K. G., Driskell, S. O., Harper S. R., Johnston, C., Browning, C., Özgün-Koca, S. A., & Kersaint, G. (2009). Mathematics teacher TPACK standards and development model. *Contemporary Issues in Technology and Teacher Education* [Online serial], *9*(1). Retrieved from http://www.citejournal.org/vol9/iss1/mathematics/article1.cfm

Otto, A., Caldwell, J., Lubinski, C., & Hancock, S. (2011). *Developing essential understanding of multiplication and division for teaching mathematics in grades 3-5*. Reston, VA: NCTM.

Petit, M., Laird, R. E., & Marsden, E. L. (2010). *A focus on fractions: Bringing research to the classroom.* New York: Taylor & Francis

Pólya, G. (1957). *How to solve it: A new aspect of mathematical method.* Princeton, NJ: Princeton University Press.

Rakes, C. R. (2010). Misconceptions in rational numbers, probability, algebra, and geometry. *Dissertation Abstracts International – A, 71*(08), Feb 2011. (UMI No. AAT 3415205) Retrieved from http://proquest.umi.com.echo.louisville.edu/pqdweb?did=2097967941&sid=1&Fmt=2&clientId=9580&RQT=309&VName=PQD

Rittle-Johnson, B., & Alibali, M. W. (1999). Conceptual and procedural knowledge of mathematics: Does one lead to the other? *Journal of Educational Psychology, 91*, 175-189.

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology, 93,* 346-362.

Ronau, R. N., & Rakes, C. R. (2011). Aspects of teacher knowledge and their interactions: A comprehensive framework for research. In R. N. Ronau, C. R. Rakes, & M. L. Niess (Eds.), *Educational technology, teacher knowledge, and classroom impact: A research handbook on frameworks and approaches* (pp. 59-102). Hershey, PA: IGI Global. DOI: 10.4018/978-1-60960-750-0.ch004

Ronau, R. N., & Rakes, C. R. (2011). Making the grade: Reporting educational technology and teacher knowledge research. In R. N. Ronau, C. R. Rakes, & M. L. Niess (Eds.), *Educational technology, teacher knowledge, and classroom impact: A research handbook on frameworks and approaches* (pp. 323-332). Hershey, PA: IGI Global. DOI: 10.4018/978-1-60960-750-0.ch014

Rudasill, K. M., Gallagher, K. C., & White, J. M. (2010). Temperamental attention and activity, classroom emotional support, and academic achievement in third grade. *Journal of School Psychology, 48,* (113-134).

Saul, M., Assouline, S., & Sheffield, L. J. (Eds.) (2010). *The peak in the middle: Developing mathematically gifted students in the middle grades*. Reston, VA: NCTM, National Association of Gifted Children, & National Middle School Association.

Schoenfeld, A. H. (1985, April). *Students' beliefs about mathematics and their effects on mathematical performance: A questionnaire analysis.* Paper presented at the annual meeting of the American Educational Research Association, Chicago, IL.

Schoenfeld, A. H. (1989). Explorations of students' mathematical beliefs and behavior. *Journal for Research in Mathematics Education*, 20, 338-355.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-371). Reston, VA: National Council of Teachers of Mathematics.

Secada, W. G. (1992). Race, ethnicity, social class, language, and achievement in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 623-660). Reston, VA: National Council of Teachers of Mathematics.

Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics, 22,* 1-36.

Shaughnessey, J. M., & Bergman, B. (1993). Thinking about uncertainty: Probability and statistics. In P. S. Wilson (Ed.), *Research ideas for the classroom* (pp. 177-197). Reston, VA: National Council of Teachers of Mathematics.

Shaughnessy, J. M., & Burger, W. F. (1985). Spadework prior to deduction in geometry. *Mathematics Teacher, 78,* 419-428.

Shrewsbury, C. M. (1993). What is feminist pedagogy? *Women's Studies Quarterly, 3-4,* 8-15.

Silver, H. F., Brunsting, J. R., & Walsh, T. (2008). *Math tools grades 3-12: 64 ways to differentiate instruction and increase student engagement*. Thousand Oaks, CA: Corwin Press.

Silver, H., Strong, R., & Perini, M. (1997). Integrating learning styles and multiple intelligences. *Educational Leadership, 55,* 22-27.

Skemp, R. R. (1976/2006). Relational understanding and instrumental understanding. *Mathematics Teaching, 77,* 20-26. Reprinted in *Mathematics Teaching in the Middle School, 12,* 88-95.

Slavin, R. E., & Karweit, N. L. (1982). *Student teams and mastery learning: A factorial experiment in urban math nine classes.* Baltimore, MD: Johns Hopkins University, Center for Social Organization of Schools. (ERIC Document Reproduction Service No. ED215904)

Slavin, R. E., & Lake, C. (2008). Effective programs in elementary mathematics: A best-evidence synthesis. *Review of Educational Research, 78,* 427-515.

Smith, J. P., diSessa, A. A., & Roschelle, J. (1993). Misconceptions reconceived: A constructivist analysis of knowledge in transition. *Journal of the Learning Sciences, 3,* 115-163.

Small, M. (2009). *Good questions: Great ways to differentiate mathematics instruction*. Reston, VA: NCTM.

Smith, M. S., & Stein, M. K. (2011). *5 Practices for orchestrating productive mathematical discussions*. Thousand Oaks, CA: Corwin Press

Smith, R. J. (2003). Equal arcs, triangles, and probability. *Mathematics Teacher, 96,* 618-621.

Sowder, J. (1991). *Understanding as a Basis for Teaching: Mathematics and Science for Prospective Middle School Teachers. Final Report.* San Diego State University: Center for Research in Mathematics, Science, & Education. (ERIC Document Reproduction No. ED339590) Retrieved from http://www.eric.ed.gov/PDFS/ED339590.pdf

Stemler, S. E., Elliott, J. G., Grigorenko, E. L., & Sternberg, R. J. (2006). There's more to teaching than instruction: Seven strategies for dealing with the practical side of teaching. *Educational Studies, 32,* 101-118.

Storeygard, J. (2009). *My kids can: Making math accessible to all learners K-5*. Portsmouth, NH: Heinemann.

Strong, R. W., Perini, M., Silver, H., & Thomas, E. (2004). Creating a differentiated mathematics classroom. *Educational Leadership, 61,* 73-78.

Strong, R. W., Silver, H. F., & Perini, M. J. (2001). Making students as important as standards. *Educational Leadership, 59,* 56-61.

Strutchens, M. (1995). *Multicultural mathematics: A more inclusive mathematics. ERIC Digest*. Washington D. C.: Office of Educational Research and Improvement

Suh, J. (2007). U.C.A.R.E. about mathematics: Assessing mathematical proficiency [online]. http://mason.gmu.edu/~jsuh4/teaching/ucareRubric.pdf

Swanson, H. L. (1990). Influence of metacognitive knowledge and aptitude on problem solving. *Journal of Educational Psychology, 82,* 306-314.

Swindal, D. N. (2000). Learning geometry and a new language. *Teaching Children Mathematics, 7,* 246-250.

Tenenbaum, G. (1986). The effect of quality of instruction on higher and lower mental processes and on the prediction of summative achievement. *Journal of Educational Research, 80,* 105-114.

Torigoe, E., & Gladding, G. (2006). Same to us, different to them: Numeric computation versus symbolic representation. *AIP Conference Proceedings, 883,* 153-156.

Usher, E. L. (2009). Sources of middle school students' self-efficacy in mathematics: A qualitative investigation. *American Educational Research Journal, 46,* 275-314.

Van Hiele, P. M. (1959/1984a). A child's thought and geometry. In D. Fuys, D. Geddes, & R. Tischler (Eds.), *English translation of selected writings of Dina Van Hiele-Geldof and Pierre M. Van Hiele* (pp. 243-252). Washington, D. C.: National Science Foundation. (ERIC Document Reproduction Service No. ED 287697)

Van Hiele, P. M. (1959/1984b). English summary by Pierre Marie Van Hiele of "The problem of insight in connection with school children's insight into the subject matter of geometry." In D. Fuys, D. Geddes, & R. Tischler (Eds.), *English translation of selected writings of Dina Van Hiele-Geldof and Pierre M. Van Hiele* (pp. 237-242). Washington, D. C.: National Science Foundation. (ERIC Document Reproduction Service No. ED 287697)

Von Glasersfeld, E. (1987). Learning as a constructive activity. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 3-18). Hillsdale, NJ: Lawrence Erlbaum.

Von Minden, A. M., Walls, R. T., & Nardi, A. H. (1998). Charting the links between

mathematics content and pedagogy concepts: Cartographies of cognition. *The Journal of Experimental Education, 66,* 339-358.

Watson, J. M., Kelly, B. A., Callingham, R. A., & Shaughnessy, J. M. (2003). The measurement of school students' understanding of statistical variation. *International Journal of Mathematical Education in Science and Technology, 34,* 1-29.

Wei, F. Y. F., & Hendrix, K. G. (2009). Gender differences in preschool children's recall of competitive and noncompetitive computer mathematics games. *Learning, Media and Technology, 34,* 27-43.

White, C. W. (1974). A study of the ability of seventh-grade and eighth-grade students to learn basic concepts of probability and the relationship between achievement in probability and selected factors. (Doctoral dissertation, University of Pittsburgh, 1974). *Dissertation Abstracts International - A, 35* (04), 1969. (UMI No. AAT 7420812)

Wright, R., Martland, J., & Stafford, A. (2006). *Early numeracy: Assessment for teaching and intervention*. London: Paul Chapman Educational Publishers

Zohar, A., & Gershikov, A. (2008). Gender and performance in mathematical tasks: Does the context make a difference? *International Journal of Science and Mathematics Education, 6,* 677-693.

IES Practice Guides Relevant to Teaching Mathematics in the Elementary School

Epstein, M., Atkins, M., Cullinan, D., Kutash, K., & Weaver, R. (2008). *Reducing behavior problems in the elementary school classroom* (NCEE 2008-012). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from http://ies.ed.gov/ncee/wwc/PracticeGuide.aspx?sid=4

Designed for elementary school educators and school- and district-level administrators, this guide offers prevention, implementation, and schoolwide strategies that can be used to reduce problematic behavior that interferes with the ability of students to attend to and engage fully in instructional activities.

Recommendations

1. *Identify the specifics of the problem behavior and the conditions that prompt and reinforce it. Every teacher experiences difficulty at one time or another in trying to remedy an individual student's behavior problem that is not responsive to preventative efforts. Because research suggests that the success of a behavioral intervention hinges on identifying the specific conditions that prompt and reinforce the problem behavior (i.e., the behavior's "antecedents" and "consequences"), we recommend that teachers carefully observe the conditions in which the problem behavior is likely to occur and not occur. Teachers then can use that information to tailor effective and efficient intervention strategies that respond to the needs of the individual student within the classroom context.*

2. *Modify the classroom learning environment to decrease problem behavior. Many effective classroom-focused interventions to decrease students' problematic behavior alter or remove factors that trigger them. These triggers can result from a mismatch between the classroom setting or academic demands and a student's strengths, preferences, or skills. Teachers can reduce the occurrence of inappropriate behavior by revisiting and reinforcing classroom behavioral expectations; rearranging the classroom environment, schedule, or learning activities to meet students' needs; and/or individually adapting instruction to promote high rates of student engagement and on-task behavior.*

3. *Teach and reinforce new skills to increase appropriate behavior and preserve a positive classroom climate. We recommend that teachers actively teach students socially- and behaviorally-appropriate skills to replace problem behaviors using strategies focused on both individual students and the whole classroom. In doing so, teachers help students with behavior problems learn how, when, and where to use these new skills; increase the opportunities that the students have to exhibit appropriate behaviors; preserve a positive classroom climate; and manage consequences to reinforce students' display of positive "replacement" behaviors and adaptive skills.*

4. *Draw on relationships with professional colleagues and students' families for continued guidance and support. Social relationships and collaborative opportunities can play a critical role in supporting teachers in managing disruptive behavior in their classrooms. We recommend that teachers draw on these relationships in finding ways to address the behavior problems of individual students and consider parents, school personnel, and behavioral experts as allies who can provide new insights, strategies, and support.*

5. *Assess whether schoolwide behavior problems warrant adopting schoolwide strategies or programs and, if so, implement ones shown to reduce negative and foster positive interactions. Classroom teachers, in coordination with other school personnel (administrators, grade-level teams, and special educators), can benefit from adopting a schoolwide approach to preventing problem behaviors and increasing positive social interactions among students and with school staff. This type of systemic approach requires a shared responsibility on the part of all school personnel, particularly the administrators who establish and support consistent schoolwide practices and the teachers who implement these practices both in their individual classrooms and beyond.*

Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel, B. (2009). *Assisting students struggling with mathematics: Response to Intervention (RtI) for elementary and middle schools* (NCEE 2009–4060). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from http://ies.ed.gov/ncee/wwc/PracticeGuide.aspx?sid=2

Taking early action may be key to helping students struggling with mathematics. The eight recommendations in this guide are designed to help teachers, principals, and administrators use Response to Intervention for the early detection, prevention, and support of students struggling with mathematics.

Recommendations:
Tier 1

1. *Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.*

Tiers 2 and 3

2. *Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8. These materials should be selected by committee.*

3. *Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.*

4. *Interventions should include instruction on solving word problems that is based on common underlying structures.*

5. *Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.*

6. *Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.*

7. *Monitor the progress of students receiving supplemental instruction and other students who are at risk.*

8. *Include motivational strategies in tier 2 and tier 3 interventions.*

Halpern, D. F., Aronson, J., Reimer, N., Simpkins, S., Star, J. R., Wentzel, K. (2007). *Encouraging girls in math and science* (NCER 2007-2003). Washington, DC: National Center for Education Research, Institute of Education Sciences, U.S. Department of Education. Retrieved from http://ies.ed.gov/ncee/wwc/PracticeGuide.aspx?sid=5

The objective of this guide is to provide teachers with specific recommendations that can be carried out in the classroom without requiring systemic change. Other school personnel having direct contact with students, such as coaches, counselors, and principals, will also find the guide useful.

Recommendations:

1. *Teachers should explicitly teach students that academic abilities are expandable and improvable in order to enhance girls' beliefs about their abilities. Students who view their cognitive abilities as fixed from birth or unchangeable are more likely to experience decreased confidence and performance when faced with difficulties or setbacks. Students who are more confident about their abilities in math and science are more likely to choose elective math and science courses in high school and more likely to select math and science-related college majors and careers.*

2. *Teachers should provide students with prescriptive, informational feedback regarding their performance. Prescriptive, informational feedback focuses on strategies, effort, and the process of learning (e.g., identifying gains in children's use of particular strategies or specific errors in problem solving). Such feedback enhances students' beliefs about their abilities, typically improves persistence, and improves performance on tasks.*

3. *Teachers should expose girls to female role models who have achieved in math or science in order to promote positive beliefs regarding women's abilities in math and science. Even in elementary school, girls are aware of the stereotype that men are better in math and science than women are. Exposing girls to female role models (e.g., through biographies, guest speakers, or tutoring by older female students) can invalidate these stereotypes.*

4. *Teachers can foster girls' long-term interest in math and science by choosing activities connecting math and science activities to careers in ways that do not reinforce existing gender stereotypes and choosing activities that spark initial curiosity about math and science content. Teachers can provide ongoing access to resources for students who continue to express interest in a topic after the class has moved on to other areas.*

5. *Teachers should provide opportunities for students to engage in spatial skills training. Spatial skills training is associated with performance in mathematics and science.*

Siegler, R. S., Carpenter, T., Fennell, F., Geary, D., Lewis, J., Okamoto, Y., ... Wray, J. (2010). *Developing effective fractions instruction for kindergarten through 8th grade*: *A practice guide* (NCEE 2010-4039). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from http://ies.ed.gov/ncee/wwc/PracticeGuide.aspx?sid=15.

This practice guide presents five recommendations intended to help educators improve students' understanding of fractions. Recommendations include strategies to develop young children's understanding of early fraction concepts and ideas for helping older children understand the meaning of fractions and the computations involved. The guide also highlights ways to build on students' existing strategies to solve problems involving ratios, rates, and proportions.

Recommendations:

1. *Build on students' informal understanding of sharing and proportionality to develop initial fraction concepts.*

2. *Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.*

3. *Help students understand why procedures for computations with fractions make sense.*

4. *Develop students' conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems.*

5. *Professional development programs should place a high priority on improving teachers' understanding of fractions and of how to teach them.*

NOTE: A webinar was conducted for this practice guide on 1/18/2012. The presentation is archived at http://www.schoolsmovingup.net/cs/smu/view/e/5167

Paschler, H., Bain, P. M., Bottge, B. A., Graesser, A., Koedinger, K., McDaniel, M., & Metcalfe, J. (2007). *Organizing instruction and study to improve student learning* (NCER 2007-2004). Washington, DC: National Center for Education Research, Institute of Education Sciences, U.S. Department of Education. Retrieved from http://ies.ed.gov/ncee/wwc/PracticeGuide.aspx?sid=1

This guide includes a set of concrete actions relating to the use of instructional and study time that are applicable to subjects that demand a great deal of content learning, including social studies, science, and mathematics. The guide was developed with some of the most important principles to emerge from research on learning and memory in mind.

Recommendations:

1. *Space learning over time. Arrange to review key elements of course content after a delay of several weeks to several months after initial presentation.*

2. *Interleave worked example solutions with problem-solving exercises. Have students alternate between reading already worked solutions and trying to solve problems on their own.*

3. *Combine graphics with verbal descriptions. Combine graphical presentations (e.g., graphs, figures) that illustrate key processes and procedures with verbal descriptions.*

4. *Connect and integrate abstract and concrete representations of concepts. Connect and integrate abstract representations of a concept with concrete representations of the same concept.*

5. *Use quizzing to promote learning. Use quizzing with active retrieval of information at all phases of the learning process to exploit the ability of retrieval directly to facilitate long-lasting memory traces.*

    5a. Use pre-questions to introduce a new topic.

    5b. Use quizzes to re-expose students to key content.

6. *Help students allocate study time efficiently. Assist students in identifying what material they know well, and what needs further study, by teaching children how to judge what they have learned.*

    6a. Teach students how to use delayed judgments of learning to identify content that needs further study.

    6b. Use tests and quizzes to identify content that needs to be learned.

7. *Ask deep explanatory questions. Use instructional prompts that encourage students to pose and answer "deep-level" questions on course material. These questions enable students to respond with explanations and supports deep understanding of taught material.*