PHILOSOPHY OF TEACHING
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Mathematics teachers are uniquely situated to influence student affect and achievement (Maher & Tetreault, 2001; Shrewsbury, 1993). To accept this responsibility, mathematics teachers must actively seek ways to understand and alleviate mathematics anxiety (Bull, 2009), evaluate and remedy underdeveloped mathematics skills, provide authentic challenges (Gulikers, Bastiaens, Kirschner, & Kester, 2008; Sluijsmans, Straetmans, & van Merriënboer, 2008), and develop connections between concepts (Hiebert & Carpenter, 1992; Skemp, 1976/2006). The National Council of Teachers of Mathematics (NCTM) outlined six principles to guide teachers toward meeting such challenges (Hiebert, 2003): Learning, Teaching, Curriculum, Technology, Equity, and Assessment (NCTM, 2000). The release of the Common Core State Standards for Mathematics (CCSSM, 2010) provided an additional framework to direct mathematics practice: (1) making sense of problems and persevere in solving them, (2) reasoning abstractly and quantitatively, (3) constructing viable arguments and critiquing the reasoning of others, (4) modeling with mathematics, (5) using appropriate tools strategically, (6) attending to precision, (7) looking for and making use of structure, and (8) looking for and expressing regularity in repeated reasoning. The NCTM Principles and the CCSSM Practice Standards offer complementary views of high quality teaching and learning of mathematics. Using these principles as a framework for inquiry (i.e., active engagement and skilled training in methods of rigorous research, leading to the development of the knowledge, skills, and dispositions necessary for critical thinking), I have sought to refine my teaching practice through continuous reflection, developing the problem solving skills necessary to incorporate research into pedagogical practice.

The Rigor and Relevance framework (Dagget & Kruse, 1999) provides a structure that helps guide my pedagogical practice (i.e., NCTM Teaching Principle). In this framework, tasks and assessment items are scaled by the degree of difficulty (Knowledge Taxonomy Model; Bloom, Englehart, Furst, Hill, & Krathwohl, 1956) and the breadth of applicability (Application Model; Daggett, 2008). These two models combine to form a robust framework comprised of four learning/performance categories: Acquisition, Application, Assimilation, and Adaptation. Activities falling within the Application and Adaptation categories build connections between mathematics content and culturally-relevant experiences while activities that fit into the Assimilation category focus on developing deep understanding of mathematics concepts, an essential tool if students are to recognize and make use of mathematical structures and patterns (CCSSM Practice Standards 7 & 8). Adaptation tasks help students develop deep understanding of mathematics concepts while connecting the mathematics to authentic experiences, providing students a pathway for moving from concrete to abstract thinking as they learn to make sense of problems (CCSSM Practice Standards 1 & 2), lending themselves well for use within a Concrete-Representational-Abstract (CRA) Sequence, as in Witzel, Mercer, & Miller (2003). These learning/performance categories provide a structure to help teachers create tasks that maximize opportunities for students to learn (Boaler, 1998), providing systematic, complex, and assessable activities for learners at different stages of proficiency. The tasks that emerge from this structure give students the opportunity to model mathematics in meaningful ways (CCSSM Practice Standard 4). Moreover, this structure helps me to hold high expectations for all students.

High expectations are not met by simply making larger/more difficult assignments or making extreme demands, creating what Hiebert and Grouws (2007) referred to as "needless frustration," "feelings of despair," "nonsensical and overly difficult problems" (p. 387). Instead, high expectations demand greater cognitive load and authentic problem solving tasks (Stein, Remillard, & Smith, 2007). Students must actively struggle with important concepts to understand mathematics in deep and useful ways (i.e., NCTM Learning Principle; Freudenthal, 1973; Hiebert & Carpenter, 1992; Hiebert & Grouws, 2007; Hoffman & Caniglia, 2009). Such active struggle must be carefully guided as students attempt to connect abstract concepts in mathematics to concrete objects through representations to avoid the development of misconceptions (Witzel et al., 2003). I design every lesson to actively engage students with both the meaning of important ideas and the connections between them, fostering procedural skills as an outgrowth of those concepts. Additionally, I seek to build connections with student values, preferences, and learning styles (Gardner, 1987, 1989; Gardner & Hatch, 1989; Silver, Strong, & Perini, 1997). Fostering a supportive, encouraging environment for students is a critical component of such practice if the struggle is to produce deep learning and an ability to persevere in mathematics problem solving (CCSSM Practice Standard 1). In my classroom, I use collaboration as a tool to create such an environment. Modeling my reasoning aloud (Kramarski and Mevarech, 2003) allows me to demonstrate how I construct my arguments and avoid erroneous reasoning. It also presents students with an opportunity to critique my reasoning before they are ever asked to construct their own reasoning, making me the target of any criticism rather than students who may be struggling. Such discussions also allow me to help students develop precise mathematical language (CCSSM Standard 6) and to teach them appropriate ways of critiquing each other (CCSSM Practice Standard 3). Once my students have had these opportunities, they tackle unfamiliar tasks that involve both routine and non-routine ways of applying the relevant mathematics, individually and in groups, constructing their own arguments and critiquing the reasoning of others (CCSSM Practice Standard 3). By providing opportunities for such cognitive struggle through targeted formative and summative assessments with careful attention to the representations used to connect abstract ideas with concrete objects (i.e., NCTM Assessment Principle), I seek to make my classes authentic, dynamic, safe, learning communities.

Incorporating multiple ways of knowing (especially across gender, race, and socio-economic backgrounds), maintaining high expectations and strong support for every student (i.e., NCTM Equity Principle), and creating authentic learning communities are also important lenses that guide my teaching (Gulikers et al., 2008; Sluijsmans et al., 2008). The CRA instructional sequence is one technique that allows students to learn mathematics through multiple modes (e.g., visual, auditory, kinesthetic, tactile) and to personalize connections between the concrete and abstract through the use of a variety of manipulatives (Witzel, Riccomini, & Schneider, 2008). I recognize that a technique/explanation that "works" in one setting with a particular group of students may not necessarily work with other sets of students in the same or different settings. Therefore, reflecting on prior lessons is a critical component of tailoring future lessons for me. As a result, each time I teach a lesson, it may differ slightly from previous versions. Since classes, like individuals, tend to learn at different rates (not necessarily linear), different sections of one of my courses are likely to be at different places in the schedule. Such latitude fosters a positive learning environment, which may be a critical component to alleviating mathematics anxiety (Maher & Tetrault, 2001; Shrewsbury, 1993). For such practice to reach its full potential, each lesson must be part of a clearly defined, coherent curriculum such as the Common Core State Standards (i.e., NCTM Curriculum Principle), incorporating appropriate tools (CCSSM Practice Standard 5) for inquiry based learning such as physical and virtual manipulatives (Fuson & Briars, 1990; Suh & Moyer, 2007), computer software tutorials and applications (Kuhn, Hoppe, Lingnau, & Wichmann, 2006), dynamic geometry software, and graphing calculator features (i.e., NCTM Technology Principle).

To summarize, the NCTM Principles and the CCSSM Practice Standards guide how I apply techniques such as the CRA sequence in my teaching. For example, these frameworks highlight the importance of encouraging students to move beyond concrete ways of thinking about mathematics and moving into abstract reasoning. On the other hand, beginning instruction at the abstract level runs the risk of neglecting the needs of some students. The use of the Rigor and Relevance framework to guide task and assessment development, think aloud discussions, and collaborative learning are some of the tools I use to help me reach the goals outlined by these principles and standards. Structuring my classes to actively engage students in learning through authentic applications that utilize technology tools where appropriate, I design assessment to form an integral component of the instruction and learning process. By using these frameworks to incorporate practices based on evidence from research, I seek to create an environment that allows all students to learn in unique, meaningful ways.

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